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NESTED REINFORCEMENT ASSEMBLIES FOR LAMINATED COMPOSITE SHELLS

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Abstract—A nested assembly of axisymmetric nets serves as an analytical model of the reinforcement of a laminated composite shell constructed of preimpregnated braided sleeves placed one within another. As an alternative to the conventional method of construction utilizing a mold or mandrel, many geometric shapes may be obtained by applying appropriate surface and edge loads to the prepreg sleeve assembly. Implementing this approach (statically induced geometry) requires a statical-geometric analysis of nested assemblies where all of the sleeve components share one surface load, but are subjected to independent edge tension forces. Depending on the particular arrangement, the individual sleeves of an assembly are treated as either geodesic or Chebyshev nets, with their respective analytical models presented in the same modular format. A computer program has been written for the analysis of an arbitrary assembly involving the two types of nets and meridional reinforcing fiber arrays; the program runs on the Macintosh II.

INTRODUCTION

The use of preimpregnated unidirectional fiber-reinforced tapes and woven fabrics in the construction of laminated composite plates and shallow shells is quite natural. Much in the same way, fiber reinforcement in the form of a nested assembly of braidings (braided sleeves) is appropriate for composite shells with a surface closed in the hoop direction (shapes such as barrel-, nozzle-, or dome-like) or in both the hoop and the longitudinal directions (shapes typical of a pressure vessel). A generic component of this reinforcement is a sleeve braided from flat fiber strands (tows). An important geometric property of a braided sleeve is that it smoothly applies to (can be "worked" or "dressed" on) any closed surface. On a surface of revolution, a braided sleeve forms an axisymmeric net with a varying net angle; in fact, it is the variability of the net angle that ensures the sleeve compliance to a given surface. Taking into account this geometric adaptation feature, the original braiding diameters of the nested sleeves can always be selected so as to satisfy the strength aspect of a composite shell design (the required directional lay-up pattern and the fiber volume of the reinforcement in the composite material).

On the other hand, in the case of a closed-surface shell, the conventional construction method (assembling unidirectionally reinforced tapes or woven fabric plies on a mandrel) leads to irregular lay-up patterns with extensive staggered splicing; a representative design and manufacturing method and, more importantly, their combined effect on the strength of composite shells has been discussed in detail by Groves (1991). Thus, for closed-surface shells, the nested sleeve layout has a natural advantage of avoiding the reinforcement splicing and its wasteful overlap.

From the manufacturing point of view, an important advantage of the nested sleeve construction is avoiding the most time-consuming and least repeatable manual operations inherent in the tape and woven fabric lay-up. As a result, the nested sleeve reinforcement facilitates both the efficient utilization of the fiber strength and the geometric consistency of laminate lay-up, thereby leading to a higher overall quality of manufacturing.

Obviously, a nested sleeve reinforcement can be utilized in a conventional way, by pulling the sleeves on a mandrel. However, mandrel disposal or removal for repeated use

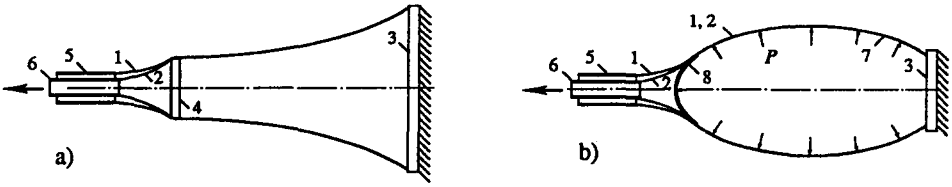

Fig. 1. Nested sleeve assembly: (a) under axial tension; (b) under axial tension and a distributed surface load.

may be difficult. Taking full advantage of the sleeve kinematics, it is possible in many cases to forego the use of mandrels and to obtain a variety of geometric shapes by statical means, i.e. by applying suitable surface and edge loads to the uncured reinforcement assembly (the concept of statically induced geometry).

Figure 1(a) represents a tensioned assembly of nested sleeves in the construction of a laminated anticlastic (nozzle-shaped) composite shell; for clarity, only two sleeve layers are shown. At one edge of the shell, all of the sleeves (1, 2) share one edge grip (3). At the other edge, the sleeves are passed over a spacer ring (4), assuring the compactness of the shell cross-section, and are individually attached to coaxial tubular grips (5, 6) which can be pulled on independently of one another. As a result of this tension, the assembly acquires a certain equilibrium configuration which becomes permanent upon matrix curing. The configuration is determined by the pertinent statical-kinematic interrelations involving the geometric parameters and tension levels of the individual sleeves.

Figure 1(b) shows a reinforcing assembly for a laminated synclastic (convex) shell. It differs from the one in Fig. 1(a) in that its shape is obtained by either using a mandrel (7) or by applying some distributed surface load. The load may be produced by diverse means, such as pressurization of a flexible bladder insert (7) or a centrifugal force field induced by spinning the entire assembly about its axis. The spacer ring (3) of the previous design is replaced by a permanent cap (8) incorporated into the shell structure. During construction and curing, the cap is supported by the mandrel or the pressurized bladder. If both edges of the shell must be closed, a similar cap, which may be fitted with a valve, is installed at the other edge.

In addition to its geometric, shape inducing effect, tensioning of the nested sleeve assembly prior to matrix curing leads to an important statical advantage. Upon curing and unloading, the composite shell acquires a residual stress (prestress) with tension in the reinforcing fibers and compression in the matrix. This is highly beneficial for the strength, stiffness and fracture toughness of the shell, especially in the case of a high-strength matrix material which may be brittle.

The objective of this paper is to develop an analytical basis for the design and analysis of nested reinforcement assemblies for composite shells, primarily within the context of statically induced geometry.

TECHNICAL BACKGROUND AND PROBLEM STATEMENT

A braided sleeve can be modeled analytically as an axisymmetric net, i.e. an assembly of two counterwound arrays of fibers located in one surface and arranged with infinitely fine spacing. At any cross-section of an axisymmetric net, a unit polar angle contains a certain constant number of fibers of each array of the net. If the fiber intersections are not fastened, in-surface slippage of the fibers may be possible. However, even with the intersections fastened, a net is a geometrically variant, multi-degree-of-freedom system, generally possessing kinematic mobility. Only certain singular configurations, called quasi-variant, are lacking kinematic mobility; such configurations are characterized by the statical possibility (but not necessarily the actual presence) of prestress.

When analysing a braided sleeve, two types of nets are relevant: Chebyshev and geodesic. The interrelated statics and geometry of these nets have been studied rather well, especially for surfaces of revolution where many closed-form solutions are available. Following is the necessary information on the geometric and statical properties of the two nets.

Chebyshev nets

All of the elementary cells in a Chebyshev net are rhombuses with one and the same length of the side. The net has been thoroughly studied in structural mechanics (Rivlin, 1959; Pipkin, 1980, 1984). The geometric theory of this net is a by-product of Chebyshev's work (for a textile industrialist) on the optimization of cutout patterns in clothes design. Chebyshev proved that due to, and at the expense of, the varying net angle, the net is applicable without folds or wrinkles to any smooth surface. In the process of applying the net to a surface, only rotation of the fibers about the surface normal is required, whereas fiber slippage at the intersections is unnecessary. Moreover, as a rule, the net will remain Chebyshev (i.e. preserve its rhombical cells) only in the absence of fiber slippage. An axisymmetric Chebyshev net obeys the equation

$$r = r_{\max} \sin \beta, \tag{1}$$

where β is the angle between one of the net lines and the surface meridian; $r_{\max}$ is the maximum theoretically possible radius of the net, attained at $\beta = \pi/2$; the smallest theoretical radius is zero and corresponds to $\beta = 0$ (net striction). Note that $r_{\max}$ is the only invariant parameter characterizing an axisymmetric Chebyshev net; both the current radius, r, and the net angle, 2β, are variables. A Chebyshev net can have a constant net angle (i.e. be isogonal, in particular, orthogonal) only on developable surfaces; on a doubly curved surface, this net always has a varying net angle.

Geodesic nets

Both arrays of a geodesic net are directed along geodesic lines which are the shortest and "straightest" lines on a surface. On a surface of revolution given by $r = r(z)$, a geodesic line is inclined to the surface meridian under an angle β such that

$$r_{\min} = r \sin \beta. \tag{2}$$

Here $r_{\min}$ is the minimum radius of revolution along the given geodesic; this radius is attained at $\beta = \pi/2$ and may be outside of the given net segment.

Only on developable surfaces can a net be simultaneously Chebyshev and geodesic; a braided sleeve with two arrays of counterwound helical strands forms just such a net on a cylindrical surface (here the helical trajectories are geodesic lines).

A braided sleeve

Kinematic mobility of a braided sleeve associated with the variability of the net angle assures that the sleeve applies smoothly to an arbitrary surface (not necessarily axisymmetric) at the expense of fiber rotations alone. However, since the fiber tow intersections in a braided sleeve are not fastened, tow slippage is not precluded. If it takes place, say, as a result of tension, it produces an in-surface geometric reconfiguration of the net, and the latter ceases to be Chebyshev. Ultimately, if in-surface slippage of the fiber tows were totally unhampered in both the longitudinal and transverse directions, the tensioned sleeve would attain a geodesic pattern and an isotensoid (uniformly tensioned) stress state. The geodesic pattern results from tension forces prompting the individual tows to shift over the surface so as to approach the shortest (geodsic) trajectories; the isotensoid state is due to the force-leveling effect of the longitudinal slippage of the tows.

Thus, with completely unimpeded tow slippage, a braided sleeve "worked" on a mandrel and tensioned would become a geodesic net described by eqn (2), whereas at the opposite extreme (no slippage anywhere), the same sleeve remains an axisymmetric Chebyshev net obeying eqn (1).

Note, however, that both of these conclusions are based on the assumption of an unlimited and unrestrained variability of the net angle. In a real sleeve, braided at a certain angle β, usually 30° or 45°, the feasible range of variation of the net angle is limited and usually is not well defined. The reason is that it depends on such diverse factors as fiber tow width and thickness, braid density (tightness), friction at the tow intersections, sleeve tension level, viscous resistance of the uncured matrix, especially, interlaminar resistance, shape of the mold, the particular techniques of "working" the mold, and so on. Pulling on the edges and vibration enhance mobility of the sleeve but, with the net angle approaching a certain minimum value, so-called tow jamming, or locking, at the intersections occurs which cannot be overcome without damaging the sleeve.

Statically induced geometry

An approach based on the concept of statically induced geometry in underconstrained structural systems (Kuznetsov, 1991) enables, in principle, a composite shell of any shape to be constructed without using a mold or mandrel. In any given configuration, an underconstrained system, in particular, a net, can balance only special, statically possible, loads (called equilibrium loads). There is no one-to-one relation between equilibrium loads and configurations: There are many different equilibrium loads for a given configuration of a system, whereas for a given load there is only one statically possible configuration (the stable equilibrium configuration). For a net, an equilibrium load producing a required geometric shape can be comprehended as an edge tension and the resulting contact surface pressure developed by the net if it was applied to the correspondingly shaped mold. Remarkably, for a given shape of the mold, every possible tension pattern at the edge of the net produces a distinct distribution of the contact surface pressure. This observation illustrates the multiplicity of equilibrium loads for a given configuration of a net and underlies a method of obtaining diverse geometric forms of nets by statical means, i.e. by utilizing a suitable combination of surface and edge loads.

Even under an edge tension alone, a variety of equilibrium shapes can be obtained. A simple, yet interesting example (Kuznetsov, 1982) is a segment of an axisymmetric Chebyshev net (like a grocery stringbag) tensioned between two parallel coaxial rings. The net acquires the form of a hyperbolic, parabolic, or elliptic pseudosphere [Figs 2 (a, b and c)], depending, respectively, on whether the meridian slope, θ (Fig. 3) is greater, equal to, or smaller than, the angle β at the same location (the relation between the two angles holds throughout the net). Thus, a variety of geometric shapes corresponding to different axial segments of each of the three surfaces is obtainable using just one sleeve and a simple mechanical fixture (tension grips). An even greater variety of nozzle-like shapes is feasible for assemblies involving several independently tensioned, interacting nested sleeves.

Surface shapes obtained by means of axial tension alone, are of negative total (Gaussian) curvature, i.e. are hourglass shaped. Producing a convex surface by statical means requires, in addition to axial tension, some kind of distributed surface load. The surface loads can be realized, for example, as pneumatic pressure (by pressurizing a flexible bladder insert); as hydrostatic pressure (by filling the bladder with a liquid); or as a centrifugal force field induced by spinning the assembly about its axis. The edge tension forces are to be exerted simultaneously but, in general, separately on each sleeve, using individual grips. By combining the surface loads with independently controlled tension forces in the nested sleeves, a wide variety of equilibrium configurations, both convex and hourglass shaped, is

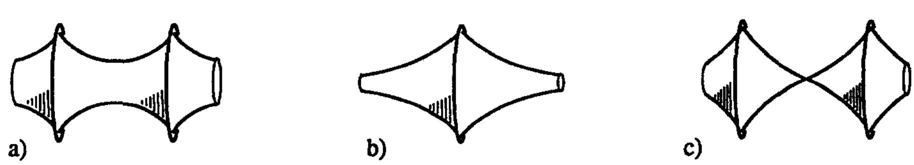

Fig. 2. The equilibrium shape of a tensioned axisymmetric Chebyshev net is one of the three types of a pseudosphere: (a) hyperbolic; (b) parabolic; (c) elliptic.

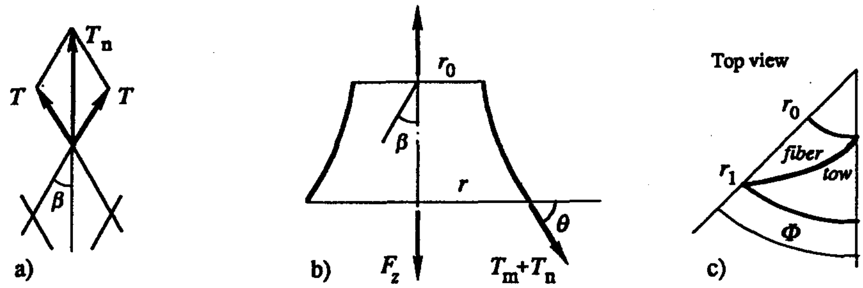

Fig. 3. Statical and geometric parameters of a nested axisymmetric assembly: (a) net forces and angle β; (b) meridional force in the assembly per unit polar angle and its axial resultant; (c) the winding angle, Φ, of a fiber tow.

obtainable. After curing the assembly, its equilibrium configuration becomes the permanent shape of the produced laminated composite shell.

Note that within the context of statically induced geometry, only the pattern of an equilibrium load is important, whereas its magnitude is irrelevant. However, the load magnitude strongly and favorably affects the mechanical characteristics of the produced composite shell. First, sleeve tensioning leads to a more uniform engagement of the fibers by reducing or eliminating their slacks and misalignments present in manually worked prepregs. Second, interlaminar pressure among the nested layers is helpful in reducing matrix voids and improving the fiber volume of the composite shell, the effects usually attained by means of vacuum cure in an autoclave. Third, upon matrix curing and unloading, the composite shell acquires a residual stress (prestress) with tension in the reinforcing fibers and compression in the matrix, which is highly beneficial for the strength, stiffness and fracture toughness of the shell.

Finally, the individual nested sleeves and their stacking sequence can be designed such that the residual stress state has a compressive transverse (interlaminar) normal stress component. For a convex shell this can be achieved by increasing the extensional stiffness and the initial tension level of the outermost sleeve relative to the respective parameters for the inner sleeves. The compressive interlaminar stress, however small in magnitude, enhances the shell resistance to delamination.

Analytical problem statement

In general, it seems appropriate to model an individual braided sleeve in a prestressed nested assembly as a Chebyshev net. However, some degree of departure from this geometry is unavoidable due to the irreversible trend for a transition of a tensioned braided sleeve towards a geodesic configuration (phenomenon known as "spreading"). The extent of the actual transformation, though usually limited, depends on the particular circumstances such as the tension level, the surface shape, the nominal (initial) sleeve diameter, tow mobility, braiding density and jamming angle, etc. Conceivably, the entire spectrum of kinematically feasible geometries, from the original Chebyshev to geodesic, can be present within one and the same sleeve. For the purpose of this analysis, the only practical way out at the moment is to postulate in advance, based on experiments, observations or reasoning, the geometric type of each sleeve component of the assembly. Specifically, three well defined geometries will be allowed: A Chebyshev net, a geodesic net, and, as a special case of the latter, a meridional array of tows uniformly spaced along the polar angle.

Upon designating the component types in the nested assembly, it can be analysed as an assembly of independently tensioned members arranged in parallel and sharing a common distributed surface load. Any number of components of each type can be assembled in an arbitrary nesting order. The adjacent layers are assumed to interact statically only in the transverse (normal) direction, and only by exerting mutual contact pressure at the interface. In kinematical terms, this assumption postulates free interlaminar slippage but prohibits delamination anywhere in the prestressed sleeve assembly. The assumption validity will be verified analytically by monitoring the normal contact stress at each interface.

The outlined approach calls for developing the necessary statical-geometric relations for the three types of components. This will be done using a modular format best suited for putting together a comprehensive model of a nested axisymmetric reinforcement assembly.

STATICAL-GEOMETRIC RELATIONS FOR COMPONENTS

Equilibrium equations

Consider an axisymmetric net subjected to a normal surface pressure, P, and an axial tension force, T_z. Let T be the tension force per unit polar angle in one fiber array [Fig. 3(a)]; recall that a unit polar angle contains a certain constant number of fibers of each array of the net. The meridional tension, T_n, in the net per unit polar angle is given by

$$T_n = 2T\cos\beta, \tag{3}$$

where β is the fiber inclination to the meridian. The total axial tension in the net is

$$T_z = 2\pi T_n \sin\theta, \tag{4}$$

with θ being the slope of the surface meridian.

The axial force T_z, which generally varies along the z-axis due to the presence of a surface pressure, is evaluated from the condition of axial equilibrium

$$2\pi Pr\,\mathrm{d}r = \mathrm{d}T_z. \tag{5}$$

Integration yields

$$T_z = T_{z0} + 2\pi\int_{r_0}^{r} P(r)r\,\mathrm{d}r = T_{z0} + 2\pi\int_{z_0}^{z} P(z)r\,\mathrm{ctn}\,\theta\,\mathrm{d}z, \tag{6}$$

where T_{z0} is the axial force at the reference parallel $z = z_0$. If pressure P is known as a function of r, the first version of this equation leads to an explicit expression for $T_z(r)$. For a uniform pressure,

$$T_z = T_{z0} + \pi P(r^2 - r_0^2). \tag{7}$$

When P is given as a function of z, the axial force cannot be obtained explicitly. In this case, choosing between z and r as an independent variable becomes a matter of computational convenience and depends on the load and support conditions of the net.

The meridional, N_1, and hoop, N_2, membrane forces (i.e. forces per unit length of the corresponding cross-sectional arcs) are related to the fiber forces as follows:

$$rN_1(=T_n) = 2T\cos\beta, \quad rN_2 = 2T\sin\beta\tan\beta. \tag{8}$$

The membrane forces appear in the equilibrium condition of the net in the direction of the normal to the net surface:

$$N_1\sigma_1 + N_2\sigma_2 = P, \tag{9}$$

where σ_1 and σ_2 are the principal curvatures of the surface. For a surface of revolution the principal curvatures can be expressed as

$$\sigma_1 = \mathrm{d}(\sin\theta)/\mathrm{d}r, \quad \sigma_2 = \sin\theta/r. \tag{10}$$

Statical-geometric relations

Combining eqns (4), (8) and (9) while taking into account (10) gives

$$T_z(\sigma_1/\sigma_2 + \tan^2\beta) = 2\pi r^2 P. \tag{11}$$

With the aid of (5) and (10), this equation reduces to

$$\frac{\mathrm{d}}{\mathrm{d}r}\frac{T_z}{\sin\theta} - \frac{T_z\tan^2\beta}{r\sin\theta} = 0, \tag{12}$$

or, in view of (4),

$$\mathrm{d}T_n/\mathrm{d}r = T_n\tan^2\beta/r. \tag{13}$$

The obtained first-order differential equation interrelates the statical and geometric parameters of equilibrium configurations of an axisymmetric net. Recall that the net is subjected to an axial force and an arbitrary normal surface pressure, although the pressure figures in the last two equations only implicitly, by means of (5). A general solution of the equation is

$$T_z/\sin\theta = 2\pi T_n = Cf, \tag{14}$$

where

$$f = f(r) = \exp\left[\int(\tan^2\beta/r)\,\mathrm{d}r\right]. \tag{15}$$

The arbitrary constant C can be evaluated in terms of either the axial force or the fiber forces in the net at any given parallel with a known θ or β. The most convenient reference parallel is the surface equator, where $\theta = 0$.

By combining $(10)_2$, (11) and (14), the normal pressure P is related to the geometric parameters of the net

$$Cf(\sigma_1 + \sigma_2\tan^2\beta) = 2\pi r P. \tag{16}$$

Solutions (14) and (16) are the sought statical-geometric relations for a general axisymmeric net.

Explicit solutions for particular types of axisymmetric nets can be obtained as soon as the angle β is specified as a function of the radius r, thereby allowing the integral (15) to be evaluated. For the two nets of interest—Chebyshev and geodesic—the respective relations between β and r are given by (1) and (2); accordingly, for a Chebyshev net

$$f = f(r) = 1/\cos\beta \tag{17}$$

and for a geodesic net

$$f = f(r) = \cos\beta. \tag{18}$$

Axisymmetric meridional array

Since meridians of a surface of revolution are geodesic lines, tension force, T_m, per unit polar angle in a meridional array is constant. The statical-geometric relations representing the respective counterparts of relations (14) and (16) are simply the equilibrium equations of a meridional fiber or tape in the longitudinal and normal directions:

$$T_z/\sin\theta = 2\pi T_m \tag{19}$$

and

$$T_m \sigma_1 = Pr. \tag{20}$$

All of the necessary statical-geometric relations for the three types of structural components of a nested axisymmetric assembly are now available. The next step is to incorporate these relations into the solving equations determining the equilibrium configurations and stress states of nested assemblies.

EQUILIBRIUM CONFIGURATIONS OF NESTED AXISYMMETRIC ASSEMBLIES

Each net component of a nested assembly is characterized by individual intrinsic (insurface) geometric parameters, like the net angle and the geodesic curvatures of the fibers. However, the extrinsic parameters, like the meridian slope and the principal curvatures of the shell surface, are the same for all of the assembly components. As a result, determining the equilibrium configuration of a nested assembly (hence, the composite shell surface) can be reduced to solving just one first-order ordinary differential equation obtainable from the above statical-geometric relations. The intrinsic parameters of each net component are evaluated in the process of integration. Two distinct cases will be considered: Tensioned assemblies without surface loads and assemblies subjected to a known surface load.

Tensioned assemblies

Summing up the individual component contributions to the total axial force of a nested assembly gives [Fig. 3(b)]

$$F_z = 2\pi(\Sigma T_m + \Sigma T_n)\sin\theta. \tag{21}$$

Here F_z is the total axial tension force in the assembly, T_m is the tension in a meridional array, T_n is the net tension introduced in (3), and the summation comprises the contributions of all of the components. Taking into account the expression for the meridian slope, $dr/dz = \text{ctn}\,\theta$, the first-order equation determining the shape of the meridian of the nested assembly is obtained

$$dr/dz = \sqrt{[2\pi(\Sigma T_m + \Sigma T_n)/F_z]^2 - 1}. \tag{22}$$

Note that both F_z and T_m are known constants given by their values at some parallel circle, whereas the meridional tension, T_n, in the net components varies along the surface axis. For each net component, the value T_n and the angle β at the initial parallel must be assigned as the initial conditions for forward integration.

A typical integration step starts with incrementing the surface radius. Knowing the type of each net component of the assembly (Chebyshev or geodesic) allows the net angle of each net to be updated with the aid of (1) or (2), and then tension T_n is updated using eqn (14) in conjunction with either (17) or (18).

According to equilibrium equation (9) as applied to a nested assembly, in the absence of normal pressure the two principal curvatures of the surface must be of the opposite signs. Thus, a prestressed nested assembly always has a negative total curvature.

Assemblies with normal surface loads

In the presence of a distributed surface load, the equilibrium configuration of the nested assembly depends on the assembly composition and the level of axial tension in the individual components. By considering the extreme cases of a very large or very small axial tension, it is obvious that even under a uniform normal pressure, the assembly surface can be concave or convex and may involve axial segments of both types. Hence, the total curvature of the surface can be of either sign and may even change its sign along the axis.

Interestingly, eqn (22) remains intact since the normal surface load is implicitly present in the component forces; however, the total axial tension in the assembly, F_z, now varies along the surface axis and must be updated at each integration step. This is done using equation (6) which, upon replacing T_z with F_z, describes axial equilibrium of the assembly; this differential equation must be integrated simultaneously with (22).

Geometric invariants of structural components

For a segment of an axisymmetric sleeve or a meridional array contained between two edge rings of fixed radii r_0 and r_1, certain geometric parameters must preserve regardless of the final equilibrium configuration of the assembly. The first invariant is the length of the fibers calculated for the final configuration of each structural component as

$$L = \int_{r_0}^{r_1} \mathrm{d}r/\cos\theta\cos\beta. \tag{23}$$

(For a meridional array, $\cos\beta = 1$.) It is assumed that the elastic elongations of the fibers can be either disregarded or evaluated independently and subtracted from the final lengths to obtain the natural (unstretched) fiber lengths.

The second invariant is the winding angle, Φ, of each fiber tow in a segment of a braided sleeve [Fig. 3(c)]. This angle is the difference between the polar coordinates of the initial and terminal points in any tow (this difference is the same for all tows in a sleeve). For a meridional array, $\Phi = 0$, while for a net the winding angle is given by

$$\Phi = \int_{r_0}^{r_1} \mathrm{d}r\tan\beta/r\cos\theta. \tag{24}$$

The unstretched fiber length and the winding angle are the only geometric invariants of each component of a nested assembly; all other parameters, including the axial length of a component, the radii of the edge parallels and even the net type, may change in the course of kinematic and elastic deformations. The significance of the two invariants is in that they uniquely determine the parameters (the nominal braiding diameter and the axial length) necessary for the actual sleeve selection. Both of the invariants are conveniently evaluated for each component in the process of forward integration.

NUMERICAL REALIZATION

The foregoing analytical development, taking advantage of the obtained closed-form first integrals for Chebyshev and geodesic nets, reduced the statical-geometric analysis of nested axisymmetric net assemblies to solving the first-order ordinary differential equation (22). A FORTRAN program employing a standard fourth-order Runge–Kutta forward integration scheme has been written and run on the Macintosh II.

Upon designating each component type (a Chebyshev net, a geodesic net or a meridional array), the following initial values are assigned at the initial parallel circle of the assembly: The radius of revolution; the meridian slope; and the axial tension in each component. The program output provides complete information on the nested assembly statics and geometry: The shape of the surface meridian (the radius of revolution as a function of the axial coordinate); the net angle for each component (needed, in particular, to assure the absence of tow jamming); fiber tension forces in each component (in a Chebyshev net the forces vary over the fiber lengths); interlaminar contact pressure (to assure the absence of delamination); the lengths and winding angles of the fiber tows (necessary for the braided sleeve selection).

At present the program allows only a trial-and-error approach to obtaining a desired geometric shape of the nested reinforcement assembly. The difficulty is the guesswork in assigning a proper set of initial data which are required for forward integration and uniquely determine the equilibrium configuration of the assembly. As a result, the shapes obtained in the initial trials of the analysis are likely to be unsatisfactory. For example, at a given axial distance from the initial edge, the surface radius will not necessarily match the required radius of the terminal edge. Furthermore, when a surface having the required axial length and edge radii is obtained, it may have an unacceptable shape. Fortunately, the computational expense of each trial run is negligible: With a compiled version of the program, its execution for any new initial data set is practically instantaneous. At the same time, observations on the evolution of the equilibrium shapes obtained in the trial runs provide a feedback for adjusting the initial conditions for the subsequent runs. Hopefully, the use of this feedback can be automated.

The ultimate objective of this work is computer-aided design of laminated composite shells shaped by statical means. The next, more challenging, step in this direction is to develop an algorithm for optimizing the selection of the braided sleeves in the nested assembly and of their tension levels so as to obtain the best feasible approximation of the required shell geometry while satisfying the strength requirements as well.

REFERENCES

Groves, S. E. (1991). Characterizing the failure of composite structures. *Mech. Engng* **113,** 57–60.
Kuznetsov, E. N. (1982). Axisymmeric static nets. *Int. J. Solids Structures* **18,** 1103–1112.
Kuznetsov, E. N. (1991). *Underconstrained Structural Systems.* Springer, New York.
Pipkin, A. C. (1980). Some developments in the theory of inextensible networks. *Q. Appl. Math.* **38,** 343.
Pipkin, A. C. (1984). Equilibrium of Tchebyshev nets. *Arch. Rat. Mech. Anal.* **85,** 81–97.
Rivlin, R. S. (1959). The deformation of a membrane formed by inextensible cords. *Arch. Rat. Mech. Anal.* **2,** 447–476.